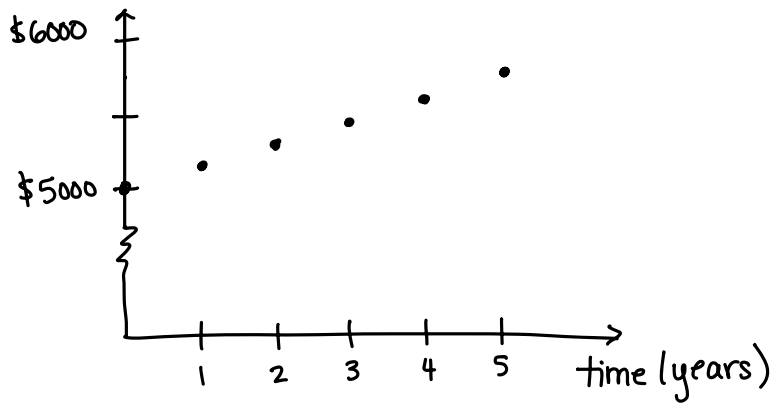
Interest

Simple Interest

- one time (informal loans)
  - borrow $5000 & agree to pay in 1 year w/ 3% simple interest
  - will pay ($5000) × (___) = $5,150 (interest $150)
- over time (bonds)
  - buy $5000 bond that pays 3% simple interest annually and matures in 5 years
    - * do not earn interest on the interest
  - will get $5000 plus 5 years of interest

    interest per year: 0.3 × $5000 = $150



future value increases _____ with time:

$5000 + (.03 × $5000) x

After 5 years (x=5): $5000 + ($150)(5) = $5750

Compound Interest

- interest on interest
  - borrow $5000, 3% interest rate, compounded annually

$5000 + (0.03)($5000) = $5000 (1.03) = $5150

$5150 + (0.03)($5150) = $5000 (1.03)² = $5304.50

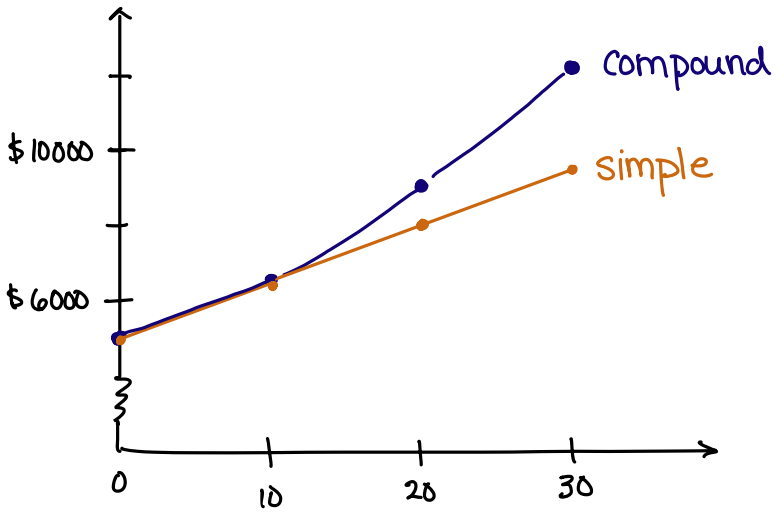
After x years: $5000 (1.03)^x

future value increases _____ with time

↳ after 5 years (x=5): $5796.37

Simple vs Compound Interest Over Time

Year	Simple	Compound
0	$5000	$5000
10	$6500	$6719.58
20	$8000	$9030.56
30	$9500	$12136.31



Subsidized vs. Unsubsidized Loans

- subsidized: government pays interest while you are in school
  - amount to pay back does not start increasing until after grace period ends
- unsubsidized: interest accrues from Day 1
  - if you pay off the interest yourself as time goes on, you don't pay interest on interest

Frequency of Compounding

Interest may be compounded every year:

$$\text{Future Value in } x \text{ years} : (\text{Initial Amt.})(1 + \text{Interest Rate})^{x}$$

Or more often, ex: quarterly, once a month, etc.

Ex $5000, 3% annual interest rate (APR), compounded quarterly

- How much interest added at end of first quarter (3 months)?
  - Divide annual interest rate by 4: .75% (0.0075)
  - Interest at end of first quarter:

    .75% of $5000: (_____) × (_____) = _____

- At end of 2nd quarter, this will contribute to _____

  .75% of $5037.50 = $38.06 (more than first quarter!)

  ↳ Excel ...

- At end of year : $5151.70 (vs $5150, comp. annually)
- What is the factor of increase over the year?

$$\frac{\square}{\$5000} = \square$$

- ... percentage increase over the year? _____

↳ This is the ____ (Annual Percentage Yield).

- APR & APY are ____ if interest is compounded _____
- APY is ____ if interest is compounded _____
  - good for investments, bad for loans

Annual Percentage Yield (APY) or Effective Rate

- Find the factor of increase over one year:

$$\frac{\text{amt. after one year}}{\text{initial amt.}}$$

- Subtract 1 to get percentage of increase over one year:

$$\text{APY} = \frac{\text{amt. after one year}}{\text{initial amt.}} - 1$$

Back to Ex: How much after x years?

Start with $5000

Increase by factor of $\left(1 + \frac{0.03}{4}\right)$

↳ do this 4 times a year for x years : $4x$ times

$\Rightarrow$ Future Value in x years : $\$5000\left(1 + \frac{0.03}{4}\right)^{4x}$

In general, future value in x years is:

$$\left(\text{Initial Amt.}\right) \times \left(1 + \frac{\text{annual interest rate}}{\#\text{ times compounded per year}}\right)^{(\#\text{times})\cdot x}$$

Abbreviated:

$$FV = P \times \left(1 + \frac{r}{n}\right)^{nx}$$

FV: future value after x years

P: initial amt. "principal" or "present value"

r = annual interest rate (APR)

n = # times per year that interest is compounded

x = # years

Interest Lab Work

Loan $8000 5.8% interest

① Calculate FV in 4 years, using tables in Excel, if interest is compounded:

(a) annually (b) quarterly (c) monthly

② Calculate FV in 4 years, using FV formula, if interest is compounded

(a) annually; compare w/ 1(a).

(b) quarterly; compare w/ 1(b).

(c) monthly; compare w/ 1(c).

(d) weekly.

(e) daily.

③ Calculate APY for each situation in #②.

For Report:

- Basic Work: Budget, Expense Tracker, Interest 1a, 2a, 3a
- Continued: Interest 1bc, 2bc, 3bc
- Further: Interest 2de
- Above and Beyond: Continuously Compounded Interest (See Canvas)